The Expressive Power of Uniform Population Protocols with Logarithmic Space

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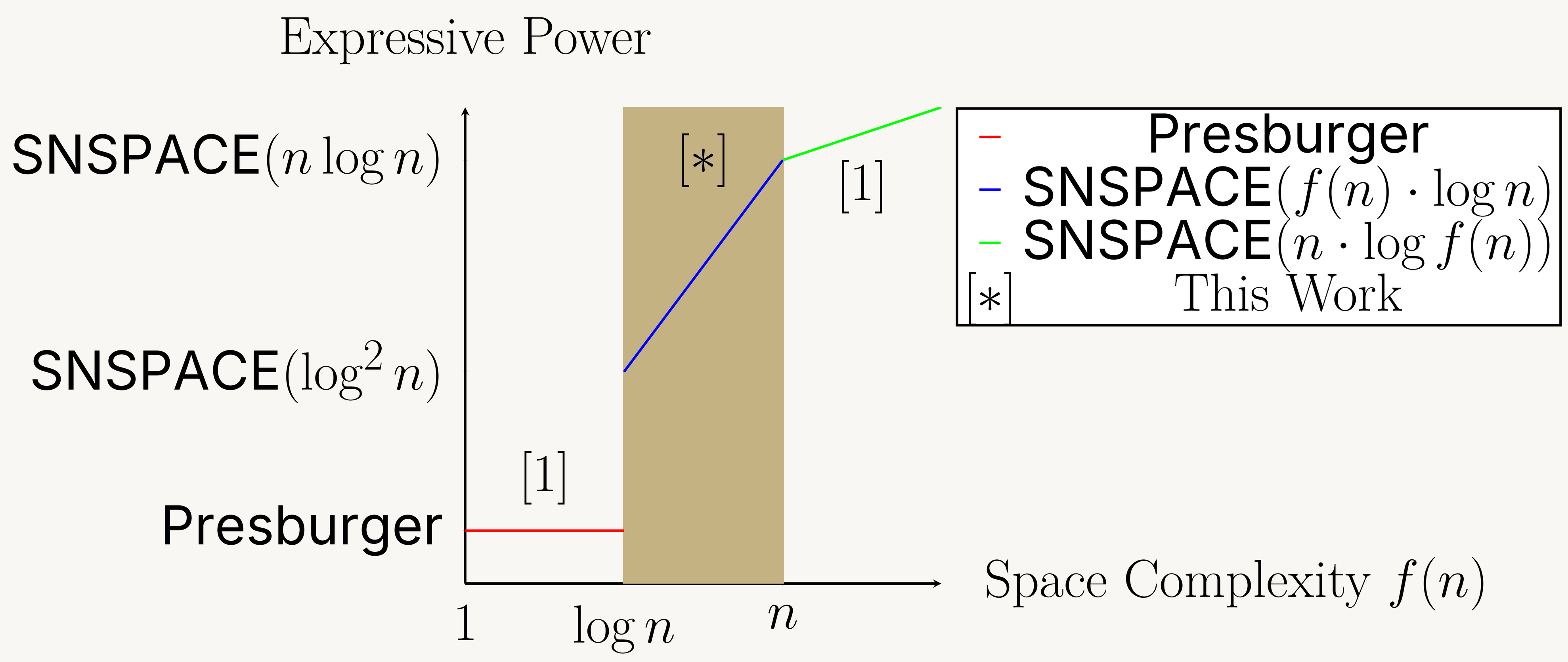
ConVeY

Population Protocols

Population protocols model distributed computation using random encounters between identical but independant agents, who can change their state during each encounter. The number of states may depend on the number of agents. A protocol is called uniform, if the transitions do not depend on the number of agents. Previously the expressive power was only known for very few ($o(\log n)$) or at least a linear number of states. We characterize the expressive power for $\Omega(\log n) \cap \mathcal{O}(n^{1-\varepsilon})$ many states.

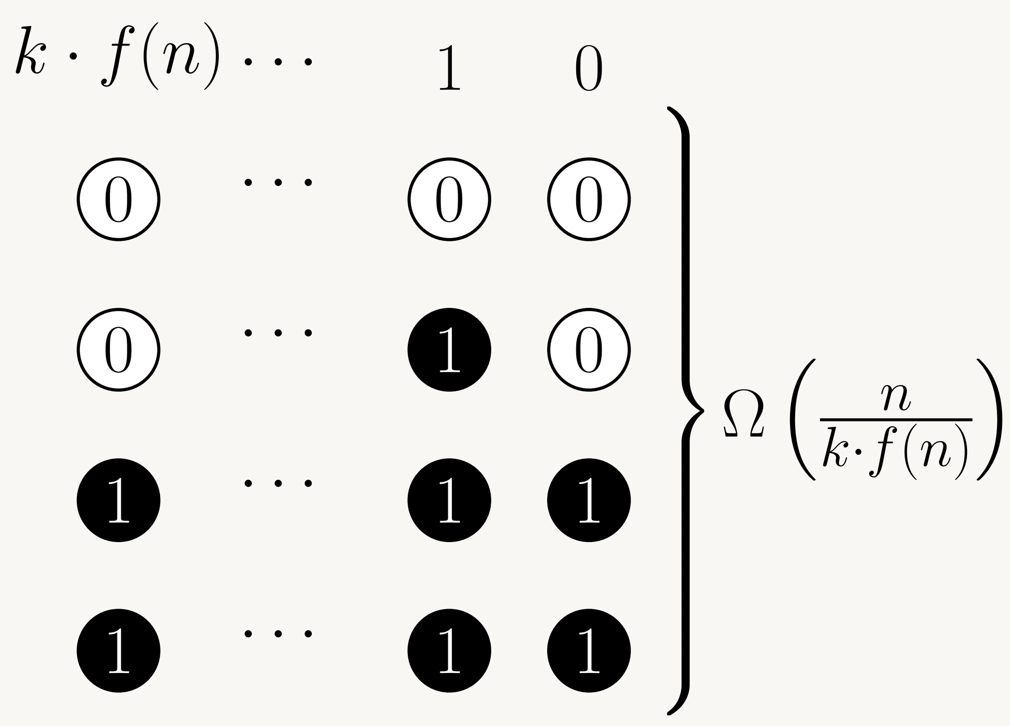
Result



Most protocols described in the literature fall within the shaded region.

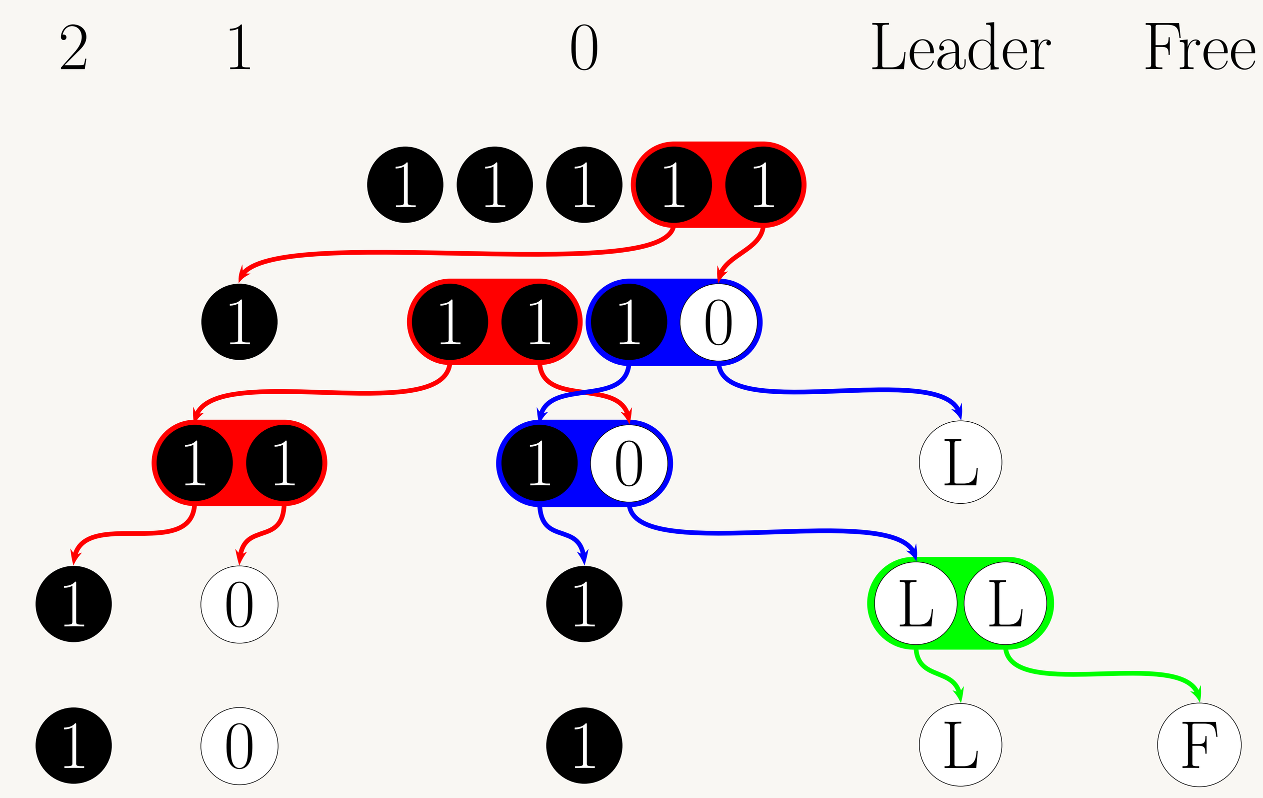
Digits

Digits simulate counters of a Counter Machine. Free agents are divided into $k \cdot f(n)$ groups, each representing a digit in base $\frac{n}{k \cdot f(n)}$. For large enough k, numbers up to $\Omega\left(2^{f(n) \cdot \log n}\right)$ can be encoded this way. Increments and decrements can be performed using for loops.



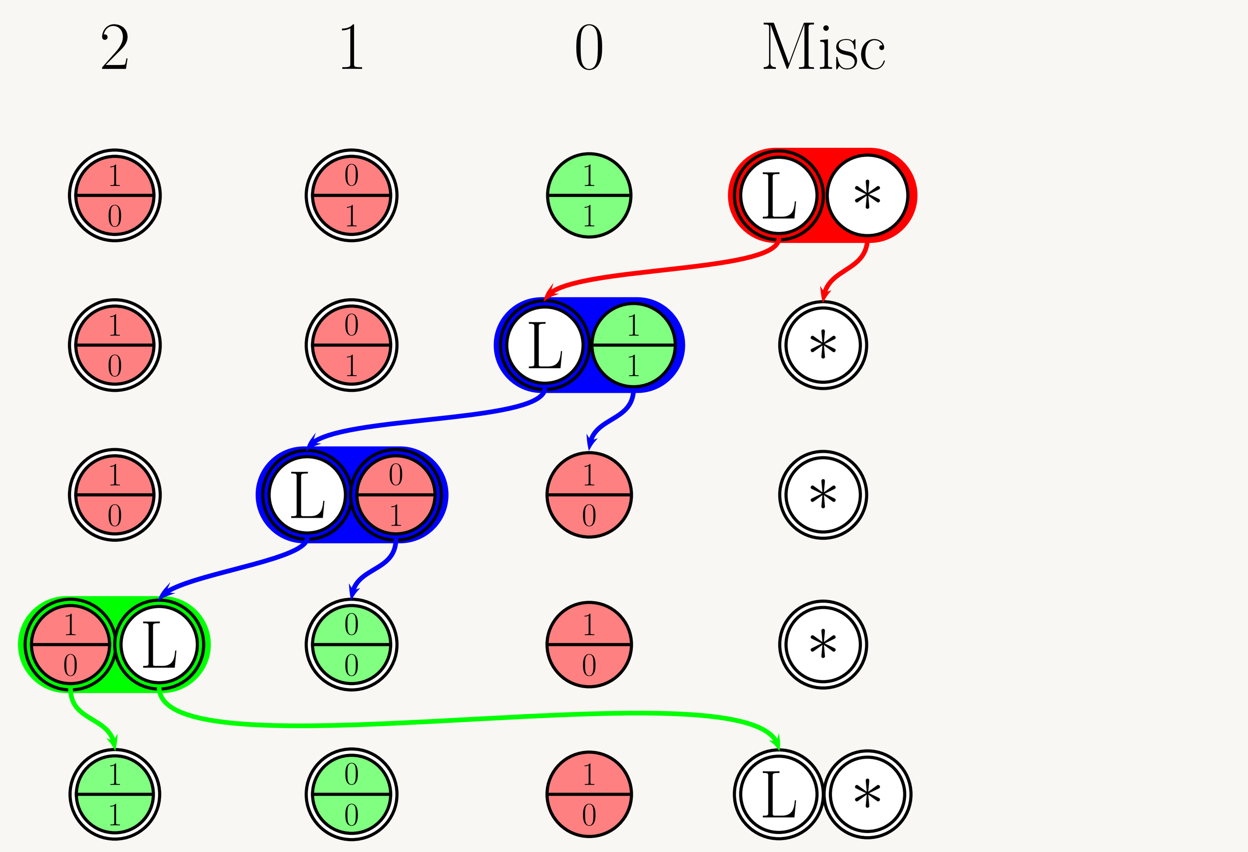
Initialisation

The number of agents is encoded in binary and a unique leader is elected.



For Loops

Counter agents store an additional bit. The leader can apply operations to all agents using a mark-and-increment strategy:



Once both counter values match, all agents have been informed.

Future Work

The expressive power of population protocols is now known for most f; however, the result from [1] for $f \in o(\log n)$ does not hold for non-uniform protocols. The expressive power of a variant model including leaders is also still open. We conjecture that the expressive power in both cases is SNSPACE$(f(n) \cdot \log n)$.

References

[1] I. Chatzigiannakis et al. "Passively mobile communicating machines that use restricted space". In: *Theor. Comput. Sci.* 412.46 (2011), pp. 6469–6483.

[2] P. Czerner et al. *The Expressive Power of Uniform Population Protocols with Logarithmic Space.* 2024. arXiv: `2408.10027 [cs.DC]`.